Conclusions

The objective of this Note was to build a method in order to estimate the aerodynamic characteristics of an axial-symmetric vehicle during reentry. The method can be considered an engineering one and is particularly suitable for feasibility study and for the early design phase. The best way to use this method is to implement all of the equations in a software such as C++ or Matlab/Simulink.

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Radiating Fin Analysis Using an Extended Perturbation Series Solution Technique

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Introduction

RADIATOR panels typically used on space vehicles to reject waste thermal energy to space commonly have efficiencies under 100% as a result of temperature gradients along the radiating surface. For thermal designers this effect (fin efficiency η) is a frequently encountered design parameter during radiator panel sizing. The fin efficiency is defined as the actual heat radiated compared to the heat radiation possible if the entire fin existed at the base temperature T_b. Several complex fin analyses have been offered in the open literature, which give designers a good estimate on the radiator panel efficiency. Kuiken[1] demonstrated that fin methods work satisfactorily for nonlinear (radiating) problems but gave nongeneral results. Aziz and Na[2] used the initial value method to investigate convecting-radiating fins, whereas Nguyen and Aziz[3] used a finite difference approach to investigate different fin profile shapes. Karem and Eby[4] linearized the fourth-power radiation term to compute fin efficiencies for the case of a constant-flux environment radiating to a sink at a temperature of 0 K. Aziz[5] employed an extended perturbation series solution to the special case of no incident heat flux on the radiating fin. His solution technique proved successful, as more fully detailed in Aziz and Na,[6] and gave designers a simple 11-term series for η, which is expanded here to include the nonhomogeneous solution of radiating to a nonzero sink. The purpose of the current study was to provide designers with similar series expansions for cases where the radiator is exchanging energy with a sink at a temperature other than 0 K. The present study can be considered as a more general case because an environmental flux can be resolved as an equivalent sink temperature, as is often the method employed by designers in radiator panel sizing calculations. Finally, for the reader interested in a general discussion of fin efficiency they can refer to Kern and Kraus,[7] who give a phenomenal description of extended surface heat transfer.

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Analysis

Equation (1) describes the one-dimensional, steady-state, and radiating extended surface temperature profile along the length of any fin as a function of the distance from the base of the fin for the case of no incident flux. This equation is valid for any rectangular fin of constant thickness w, where x is the distance along the fin from the base, A_r is the area exposed to the heat sink at temperature T_s, k is the base material conductivity, A_c is the conductive cross-sectional area, L is the length of the fin, σ is the Stefan–Boltzmann constant, and ε is the emissivity of the fin.

$$\frac{kA_c}{L}\frac{\mathrm{d}^2T}{\mathrm{d}x^2} - \sigma A_r \varepsilon\left(T^4 - T_s^4\right) = 0 \tag{1}$$

By introducing the following nondimensional parameters, as developed by Aziz and Na,[6] a simpler form of Eq. (1) is rendered:

$$\Theta = \frac{T}{T_b}, \qquad \Theta_s = \frac{T}{T_s}, \qquad X = \frac{x}{L}, \qquad \zeta = \frac{2\sigma\varepsilon T_b^3 L^2}{kw}$$

For steady state with the boundary conditions describing a fin with uniform base temperature and an insulated tip, Eq. (1) becomes

$$\frac{\mathrm{d}^2\Theta}{\mathrm{d}X^2} - \zeta\left(\Theta^4 - \Theta_s^4\right) = 0 \tag{2}$$

with the transformed boundary conditions,

$$\Theta(0) = 1, \qquad \frac{\mathrm{d}\Theta(1)}{\mathrm{d}X} = 0$$

Solution Technique and Results

The approach taken in Ref. 5 was not to search for a particular solution to the nonlinear differential Eq. (2), but rather to describe the behavior of the solution. The technique used was to perform a regular perturbation series expansion on Θ, as shown in Eq. (3):

$$\Theta = \sum_{i=0}^{\infty} \zeta^i \Theta_i \tag{3}$$

In Ref. 5 (and many others) Θ_s was taken as 0 for simplicity. Next, by substituting Eq. (3) into Eq. (2) and combining like terms in powers of ζ yielded the infinite perturbation series solution. The first 11 terms were hand calculated by Aziz in Ref. 5, giving the

Table 1 SINDA model comparison with series solution; Eqs. (5) and (6)

		Sinda results		Series expansion			
No.	Power, W	Θ_t	η	ζ	Θ_t	η	Error, %
1	2	0.924	0.773	0.211	0.920	0.804	4.0
2	2	0.934	0.801	0.175	0.931	0.829	3.5
3	4	0.888	0.723	0.375	0.878	0.714	1.2
4	4	0.904	0.776	0.311	0.893	0.745	4.0
5	10	0.810	0.599	0.861	0.799	0.560	6.5
6	10	0.830	0.637	0.690	0.821	0.603	5.3

Table 2 Series expansion terms and radii of convergence for Eq. (3)

Value of ζ	$\Theta_s = 0.00$	$\Theta_s = 0.05$	$\Theta_s = 0.10$	$\Theta_s = 0.15$	$\Theta_s = 0.20$	$\Theta_s = 0.25$	$\Theta_s = 0.30$	$\Theta_s = 0.35$	$\Theta_s = 0.40$
ζ_0	0.2571	0.2577	0.2632	0.2660	0.2717	0.2778	0.2817	0.2890	0.2959
ζ_1^*	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
ζ_2^*	−0.128535	−0.122408	−0.118440	−0.113050	−0.108680	−0.104175	−0.095095	−0.093925	−0.088770
ζ_3^*	−0.073464	−0.069833	−0.066484	−0.062931	−0.059466	−0.055942	−0.052033	−0.048684	−0.044992
ζ_4^*	−0.050766	−0.047793	−0.044896	−0.041977	−0.039063	−0.036137	−0.033408	−0.030321	−0.027415
ζ_5^*	−0.038416	−0.035786	−0.033186	−0.030634	−0.028081	−0.025551	−0.023371	−0.020653	−0.018260
ζ_6^*	−0.030690	−0.028280	−0.025894	−0.023592	−0.021301	−0.019059	−0.017219	−0.014822	−0.012801
ζ_7^*	−0.025443	−0.023169	−0.020945	−0.018831	−0.016743	−0.014724	−0.013125	−0.010997	−0.009262
ζ_8^*	−0.021661	−0.019477	−0.017383	−0.015418	−0.013494	−0.011657	−0.010242	−0.008341	−0.006834
ζ_9^*	−0.019002	−0.016695	−0.014708	−0.012865	−0.011079	−0.009394	−0.008127	−0.006419	−0.005102
ζ_{10}^*	−0.017254	−0.014528	−0.012632	−0.010893	−0.009224	−0.007670	−0.006526	−0.004985	−0.003827
ζ_{11}^*	−0.015804	−0.012797	−0.010979	−0.009330	−0.009224	−0.006323	−0.000884	−0.388968	−0.002867

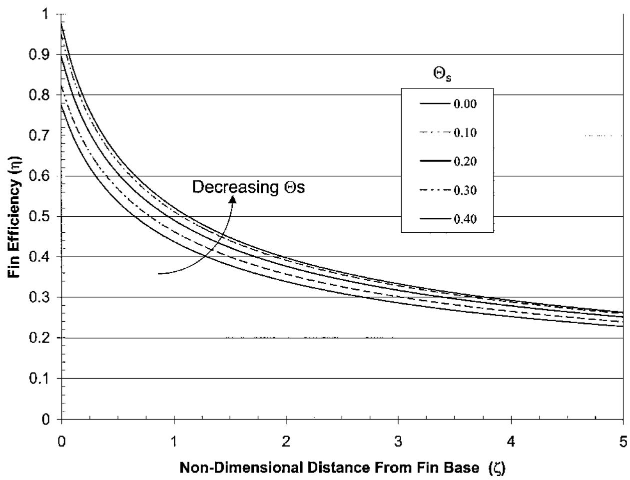

Fig. 1 Fin efficiency dependence on ζ and Θ_s from Eq. (6) using values from Table 2.

resulting series for the normalized tip temperature (Θ_t) at $X = 1$, as

$$\Theta_t = 1.000000 - 0.500000\zeta + 0.833333\zeta^2 - 1.905555\zeta^3 + 5.043055\zeta^4 - 14.514290\zeta^5 + 44.158916\zeta^6 - 140.236100\zeta^7 + 457.870867\zeta^8 - 1526.744970\zeta^9 + 5175.665448\zeta^{10} - \cdots + \cdots \tag{4}$$

The radius of convergence of Eq. (4) is approximately $\zeta_0 = 0.25707$. This estimation was obtained by Aziz using a Domb and Skyes extrapolation of d'Alembert's ratio test. Aziz and Na[6] suggested making an Euler transformation of Eq. (4), which would recast the series in powers of $\zeta/(\zeta_0 + \zeta)$ instead of ζ. The advantage of this transformation is to extend the circle of convergence to infinity. The new series, which converges for all values of ζ, becomes

$$\Theta_t = 1.000000 - 0.128535\zeta^* - 0.073464\zeta^{*^2} - 0.050766\zeta^{*^3} - 0.038416\zeta^{*^4} - 0.030690\zeta^{*^5} - 0.025443\zeta^{*^6} - 0.021661\zeta^{*^7} - 0.019002\zeta^{*^8} - 0.017254\zeta^{*^9} - 0.015804\zeta^{*^{10}} - \cdots \tag{5}$$

where $\zeta^* = \zeta/(\zeta_0 + \zeta)$.

The accuracy of this expansion was verified in Ref. 6, and the interested reader is referred there. An independent verification of the accuracy of Eq. (5) was also performed via a finite difference model using a 120-node SINDA model, and agreement was found to within 7% (Table 1).

The series solution given by Eq. (5) performed quite well when compared with a numerical difference model, and this series is easily programmed into any pocket calculator or spreadsheet and is an excellent tool for first-cut radiator hand calculations. However, to extend the initial expansion performed by Aziz[5] to include results other than for $\Theta_s = 0$ the same approach was taken to include eight additional cases covering Θ_s from 0.05 to 0.40. Table 2 includes the terms needed for the expansions along with the radii of convergence found by the Domb and Skyes plots, as was used in Refs. 5 and 6.

Finally, the fin efficiency η, in terms of Θ_t and ζ given in Ref. 7, is shown as Eq. (6). The resulting fin efficiency found from using the terms found in Table 2 to compute Θ_t from Eq. (5) for use in Eq. (6) is shown for several different values of Θ_s in Fig. 1:

$$\eta = 2\sqrt{\left(1 - \Theta_t^5\right)/10\zeta} \tag{6}$$

Conclusions

The approach taken by Aziz in Ref. 5 for fin efficiency computations is now readily expanded for cases involving a nonzero radiating heat-sink temperature. The method is simple and easily adapted using Eqs. (5) and (6) along with the values for the expansion terms in Table 2. The accuracy of the fin efficiency using the series expansion method was found to be within 7% when compared to a SINDA fin model for the zero-sink temperature case.

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